CS 188: Artificial Intelligence

Lecture 7: Utility Theory

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Many slides adapted from Dan Klein

Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility:
  - A rational agent should chose the action which maximizes its expected utility, given its knowledge
- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - Why are we taking expectations of utilities (not, e.g. minimax)?
  - What if our behavior can't be described by utilities?

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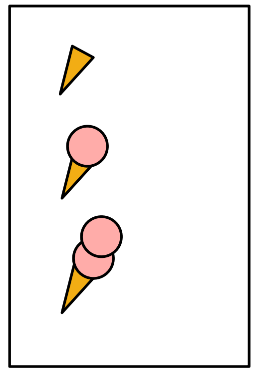
Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function
- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?



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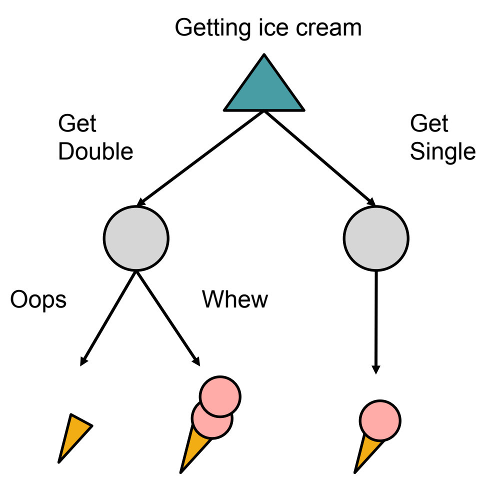
Utilities: Uncertain Outcomes



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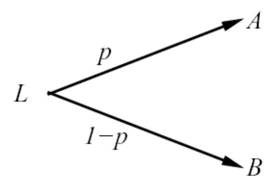
Preferences

- An agent must have preferences among:
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A;\ (1-p), B]$$



- Notation:

$A \succ B$ A preferred over B

$A \sim B$ indifference between A and B

$A \succeq B$ B not preferred over A

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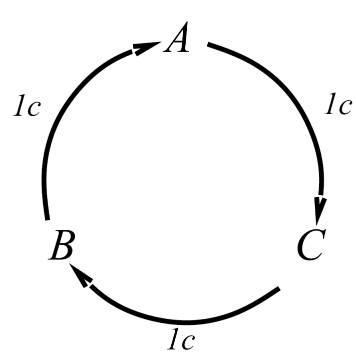
Rational Preferences

- We want some constraints on preferences before we call them rational

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C



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Rational Preferences

- Preferences of a rational agent must obey constraints.
  - The axioms of rationality:

Orderability

$(A \succ B) \vee (B \succ A) \vee (A \sim B)$

Transitivity

$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

Continuity

$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$

Substitutability

$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$

Monotonicity

$A \succ B \Rightarrow$

$(p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$

- Theorem: Rational preferences imply behavior describable as maximization of expected utility

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MEU Principle

- Theorem:
  - [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \ \Leftrightarrow\ A \succeq B$$

$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \sum_i p_i U(S_i)$$

- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tictactoe, reflex vacuum cleaner

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Utility Scales

- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

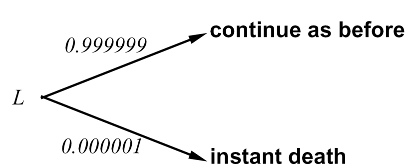
$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

Human Utilities

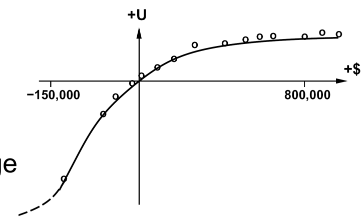
- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment of human utilities:
  - Compare a state A to a standard lottery L_p between
    - "best possible prize" u_+ with probability p
    - "worst possible catastrophe" u_- with probability 1-p
  - Adjust lottery probability p until A ~ L_p
  - Resulting p is a utility in [0,1]

pay $30 **~**



Money

- Money does not behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery L = [p, $X; (1-p), $Y]
  - The expected monetary value EMV(L) is p*X + (1-p)*Y
  - U(L) = p*U($X) + (1-p)*U($Y)
  - Typically, U(L) < U(EMV(L)): why?
  - In this sense, people are risk-averse
  - When deep in debt, we are risk-prone
- Utility curve: for what probability p am I indifferent between:
  - Some sure outcome x
  - A lottery [p,$M; (1-p),$0], M large



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Example: Insurance

- Consider the lottery [0.5,$1000; 0.5,$0]
  - What is its expected monetary value? ($500)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - $400 for most people
  - Difference of $100 is the insurance premium
    - There's an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!

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Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8,$4k; 0.2,$0]
  - B: [1.0,$3k; 0.0,$0]

  - C: [0.2,$4k; 0.8,$0]
  - D: [0.25,$3k; 0.75,$0]

- Most people prefer B > A, C > D
- But if U($0) = 0, then
  - B > A ⇒ U($3k) > 0.8 U($4k)
  - C > D ⇒ 0.8 U($4k) > U($3k)

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